PA-003-001617 Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

March / April - 2020

BSMT - 602 (A) : Mathematics

(Analysis - II & Abstract Algebra - II)

(Old Course)

Faculty Code : 003

Subject Code : 001617

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Figures to the right indicate full marks of the question.

1 Answers the following questions in short : **20**

(1) How many non unit elements in the ring $(\mathbb{Z}_4, +_4, \times_4)$?

(2) What is the multiplicative inverse of 4 in $(\mathbb{Z}_7, +_7, \times_7)$?

(3) List all the ideals of the ring $(\mathbb{R}, +, \cdot)$.

(4) Let $f : G \to G; f(g) = e(g \in G)$, be a group homomorphism, where e is the identity of G. Find ker f.

(5) True or False : Every field is an integral domain.

(6) True of False : $\{0\}$ is a connected subset of $\mathbb{R}$.

(7) Give an example of a compact metric space.

(8) Give an example of a non-commutative ring with unity.

(9) If $f = (2, 8, 2, 11, 0, 0, 0, \ldots)$ and $g = (-1, -1, 0, -2, 0, 0, 0, \ldots)$, then find $f + g$.

(10) Find $L\left(t^5\right)$.

(11) Find $L^{-1}\left(\dfrac{s^2 - \pi s + e}{s^3}\right)$.

(12) Find convolution product of $f(t) = t^2$ and $g(t) = t$.

(13) Find $L^{-1}\left(\frac{1}{s-\pi}\right)$.

(14) State First Shifting Theorem for Laplace Transform.

(15) Define : Compact metric space.

(16) Define : Kernel of group homomorphism.

(17) Define : Countable set.

(18) Define : Laplace transform.

(19) Define : Subring.

(20) Define : Ideal.

2 (A) Attempt Any **Three** : **6**

(1) Show that $\mathbb{N}$ is not a compact subset of $\mathbb{R}$.

(2) Give an example of a subset of $\mathbb{R}$ which is not connected.

(3) Define : Sequentially compact metric space.

(4) Show that $\{-1,0,1\}$ is a compact subset of $\mathbb{R}$.

(5) Find Laplace transform of $f(t) = \begin{cases} \sin t & 0 < t < \pi \\ 0 & t > \pi \end{cases}$

(6) Find $L^{-1}(F(s))$. Where $F(s) = \log\frac{s+a}{s+b}$.

(B) Attempt Any **Three** : **9**

(1) If X is a finite set, then show that (X, d) with a discrete metric is compact space.

(2) Show that $\mathbb{Q}$ is countable.

(3) Show that every compact subset of a metric space is bounded.

(4) Prove that continuous image of a compact set is compact.

(5) Find Laplace transform of $f(t) = t\cos^2 t$.

(6) Find inverse Laplace transform of $F(s) = \frac{2s+3}{s^2+2s+2}$.

(C) Attempt Any **Two** : **10**

(1) State and prove Convolution Theorem.

(2) State and prove Heine-Borel Theorem for $\mathbb{R}$.

(3) Prove that every closed subset of a compact metric space (X,d) is a compact.

(4) Find inverse Laplace transform of $F(s)=\frac{s+2}{s(s+1)(s+3)}$.

(5) Evaluate $\int_0^{\infty} e^{-3t} t \sin t \, dt$.

3 (A) Attempt Any **Three** : **6**

(1) Show that $(\mathbb{Z}_6, +_6, \times_6)$ is not an integral domain.

(2) What is the zero element of the ring $(M_2(\mathbb{R}),+,\cdot)$?

(3) Show that in an integral domain 0 and 1 are the only idempotent elements.

(4) Show that $\mathbb{Z}[\sqrt{7}]=\{a+b\sqrt{7} \mid a,b \in \mathbb{Z}\}$ is not a field under usual addition and multiplication.

(5) Does $S=\{A \in M_2(\mathbb{R}) \mid \det(A)=1\}$ is a subring of $(M_2(\mathbb{R}),+,\cdot)$? Justify.

(6) What is the characteristic of the ring $(\mathbb{Z}_4,+_4,\times_4)$?

(B) Attempt Any **Three** : **9**

(1) Prove that intersection of two ideals of a ring is also an ideal of the ring.

(2) Let I, J be ideals of a commutative ring R. Show that if $I \cup J$ is an ideal in R, then $I \subset J$ or $J \subset I$.

(3) Let I be an ideal of a commutative ring with unity. Show that if $1 \in I$, then $I = R$.

(4) Show that $(\mathbb{Z}_n, +_n, \times_n)$ is a principal ideal ring.

(5) Show that the polynomial
$g(x) = 3x^5 + 15x^4 - 20x^3 + 10x + 20$ is irreducible over $\mathbb{Q}$.

(6) Prove that the characteristic of an integral domain is 0 or prime.

(C) Attempt Any **Two** : **10**

(1) Let R be a ring with unity. Then prove that : the characteristic of R is n iff n is the smallest positive integer such that $n1 = 0$.

(2) State and prove Fundamental Theorem of group homomorphism.

(3) Prove that every finite integral domain is a field.

(4) Prove that field has no proper ideal.

(5) Let $\varphi : G \to \overline{G}$ be a group homomorphism. Show that φ is one-one iff ker $\varphi = \{e\}$.